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## ICT – ENABLED FACILITIES









$$X = \{a, b, c, d\}$$

$$T = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$A = \{b, c\} \subset X$$

$$\emptyset^c = X$$

$$X^c = \emptyset$$

$$\{a\}^c = \{b, c, d\}$$

$$\{a, b\}^c = \{c, d\} \subset X$$

$$\overline{A} = X \cap \{b, c, d\}$$

$$\overline{A} = \{b, c, d\}$$

$$\mathcal{C}(A) = \overline{A}$$

Conversely, let  $A = C \cap Y$ , for some closed subset  $C$  of  $X$ .  
 Then, we shall prove that  $A$  is closed in  $Y$ .  
 Finally, we shall prove  $Y - A = Y \cap (X - C)$ .  
 Let  $y \in Y \cap (X - C)$   
 $\Rightarrow y \in Y$  and  $y \notin C$   
 $\Rightarrow y \in Y$  and  $y \notin A$   
 $\Rightarrow y \notin C \cap Y$   
 $\Rightarrow y \notin A$   
 $\Rightarrow Y - A = Y \cap (X - C)$

Hence,  $Y - A = Y \cap (X - C)$   
 It is clear that  $Y - A$  is open in  $Y$  b/c  $(X - C)$   
 is open in  $X$ , so  $Y - A \in T$ .  
 Thus, we can say that  $A$  is closed in  $Y$ .

\* Closure of a set  $\rightarrow$  Let  $(X, T)$  be a topological space and  $A \subset X$ . The closure of  $A$  is defined by intersection of all closed supersets of  $A$ , i.e.  
 $\mathcal{C}(A) = \overline{A} = \bigcap \{C \subset X : C \text{ is closed and } A \subset C\}$

Eg:  $X = \{a, b, c\}$ ,  $T = \{\emptyset, X, \{a\}, \{b, c\}, \{c, a, c\}\}$   
 $A = \{a, b, c\}$



 **GPS Map Camera**



**Gwalior, Madhya Pradesh, India**

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474006, India

Lat 26.241608°

Long 78.228212°

27/01/24 02:05 PM GMT +05:30

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◇ METRIZABLE SPACE  $\Rightarrow$  A topological space  $(X, \mathcal{T})$  is said to be metrizable if  $\exists$  a metric  $d$  on  $X$  such that  $\mathcal{T} = \mathcal{T}_d$  where  $\mathcal{T}_d$  is a topology generated by metric  $d$ . Example: usual topology

Note: Every metric space is a Topological Space but its converse is not true.

Example: Let  $(X, \mathcal{T})$  be a Topological Space where  $X = \{a, b\}$   $\mathcal{T} = \{\emptyset, X, \{a\}\}$   $a \neq b$

○ We claim that topological space  $(X, \mathcal{T})$  is not metrizable.

If possible, suppose that  $(X, \mathcal{T})$  is metrizable, i.e.,  $\exists$  a metric  $d$  on  $X$  such that  $\mathcal{T} = \mathcal{T}_d$ , where  $\mathcal{T}_d$  is a family of all open subsets of  $X$  w.r.t  $d$ .

Put  $d(a, b) = \lambda \neq 0$

Consider two nbds  $N_{\lambda/2}(a) = \{x \in X : d(x, a) < \lambda/2\}$   
 $\Rightarrow N_{\lambda/2}(a) = \{a\}$  ( $\because$   $X$  has only two elements)

Similarly,  $N_{\lambda/2}(b) = \{b\}$

Dr. R.N.GUPTA

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$X = \{a, b\}$

$\{b\} \notin \mathcal{T} = \{\emptyset, X, \{a\}\}$

$d: \Sigma \times \Sigma \rightarrow \mathbb{R}$

$\mathcal{T} \neq \mathcal{T}_d$   $d(a, b) > 0$

$\neq$

$\{a\} \text{ --- } \{b\} \notin \mathcal{T}$

$\mathcal{T}_d = \{\emptyset, X, \{a\}, \{b\}\}$